

FLUID PRESSURE AND ITS MEASUREMENTS

Consider a small area dA in large mass of the fluid. If the fluid is stationary, then the force exerted by the surrounding fluid on the area dA will be always perpendicular to the surface dA . Let dF is the force acting on the area dA in the normal direction. Then the ratio of $\frac{dF}{dA}$ is known as the intensity of pressure.

$$P = \frac{dF}{dA}$$

If the force F is uniformly distributed over the area A . Then the pressure at any point is given by

$$P = \frac{F}{A} = \frac{\text{Force}}{\text{Area}}$$

Pressure force (F) = $P \times A$

Unit: N/m^2 , N/mm^2 , Pascal (Pa), bar, Atmospheric pressure, mm of Hg

$1N/m^2 = 1 \text{ Pa}$, $1 \text{ bar} = 10^5 \text{ N/m}^2$, $1 \text{ atm} = 101325 \text{ N/m}^2$

PASCAL'S LAW

It states that the pressure or intensity of the pressure at a point in a static fluid equal in all directions.

PRESSURE VARIATION IN A FLUID AT REST/ HYDROSTATIC LAW:

The rate of increase of pressure in a vertically downward direction must be equal to the specific weight of the fluid at that point.

Consider a small fluid element,

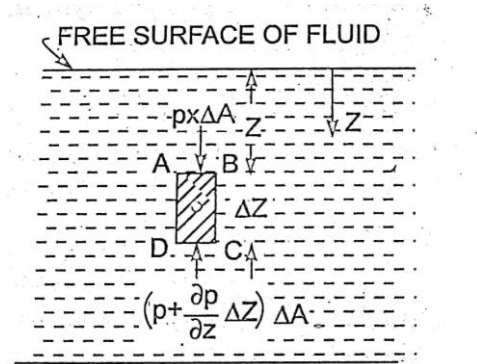
Let ΔA = cross – sectional area of the element

ΔZ = Height of fluid element

P = Pressure on face AB

Z = Distance of fluid element from free surface.

The forces acting on the fluid element are



Forces on a fluid element.

1. Pressure force on AB = $P \times \Delta A$ acting perpendicular to the face AB in the downward direction.
2. Pressure force on CD = $(P + \frac{\partial p}{\partial z} \times \Delta Z) \times \Delta A$, acting perpendicular to face CD in vertically upward direction.
3. Weight of fluid element = density $\times g \times$ volume = $\rho \times g \times \Delta A \times \Delta Z$
4. Pressure forces on the surfaces BC and AD are equal and opposite. So they cancel each other.

Under the condition of equilibrium Σ All vertical forces = 0

$$P \times \Delta A - (P + \frac{\partial p}{\partial z} \times \Delta Z) \times \Delta A + \rho \times g \times \Delta A \times \Delta Z = 0$$

$$P \times \Delta A - P \times \Delta A - \frac{\partial p}{\partial z} \times \Delta Z \times \Delta A + \rho \times g \times \Delta A \times \Delta Z = 0$$

$$-\frac{\partial p}{\partial z} \times \Delta Z \times \Delta A + \rho \times g \times \Delta A \times \Delta Z = 0$$

$$\rho \times g \times \Delta A \times \Delta Z = \frac{\partial p}{\partial z} \times \Delta Z \times \Delta A$$

$$\frac{\partial p}{\partial z} = \rho \times g = w$$

$$\partial p = \rho \times g \times \partial z$$

By integrating the above equation for liquid

$$\int \partial p = \int \rho \times g \times \partial z$$

$$P = \rho \times g \times z = w \times z$$

$$z = \frac{P}{\rho \times g}$$

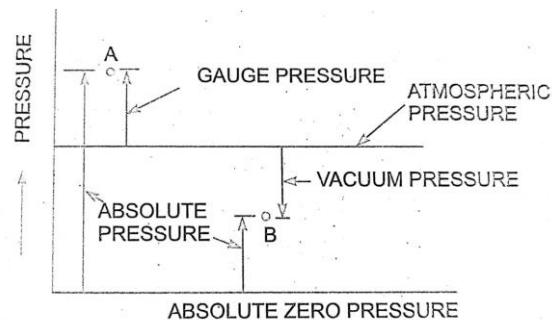
Where P is the pressure above the atmospheric pressure and z is the height of the point from free surfaces. This z is called as pressure head. It is the measure of energy in terms of meter.

ABSOLUTE PRESSURE, GAUGE PRESSURE, VACUUM PRESSURE:

The pressure on a fluid element is measured in two different systems.

In one system, it is measured above the absolute zero pressure and it is called as absolute pressure.

In other system the pressure is measured above atmospheric pressure and it is called as gauge pressure.



Absolute Pressure is defined as the pressure which is measured with reference to absolute vacuum pressure.

Gauge pressure is defined as the pressure which is measured with the help of a pressure measuring instrument, in which the atmospheric pressure is taken as datum. The atmospheric pressure on the scale is marked as zero.

Vacuum pressure is defined as the pressure below the atmospheric pressure below the atmospheric pressure.

Mathematically,

Absolute pressure = Atmospheric pressure + Gauge pressure

Vacuum Pressure = Atmospheric pressure – Absolute pressure

MEASUREMENT OF PRESSURE:

The pressure can be measured by following devices. They are

1. Manometers
2. Mechanical Gauges

MANOMETERS:

Manometers are defined as the devices that are used for measuring the pressure at a point in a fluid by balancing the column of fluid by the same or another column of the fluid. They are classified as: i. Simple Manometer, ii. Differential Manometer

MECHANICAL GAUGES:

Mechanical gauges are defined as the devices used for measuring the pressure by balancing the fluid column by the spring or dead weight. The commonly used mechanical pressure gauges are: i. Diaphragm Pressure Gauge, ii. Bourdon tube pressure gauge, iii. Dead weight pressure gauge, iv. Bellows pressure gauge.

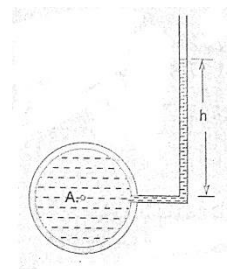
SIMPLE MANOMETER:

A simple manometer consists of a glass tube having one of its ends connected to a point where pressure is to be measured and other end remains open to the atmosphere. Common types of manometers are

1. Piezometer
2. U tube Manometer
3. Single Column Manometer

PIEZOMETER:

It is the simplest form of manometer used for measuring gauge pressure. One end of this manometer is connected to the point where pressure needs to be measured and other end is opened to the atmosphere as shown in the figure. The rise of liquid gives the pressure head at that point. If at a point A, the height of the liquid say water is h in the piezometer, then the pressure at point A = $\rho \times g \times h$



U-TUBE MANOMETER:

It consist of a glass tube bent in U shape, one end of which is connected to a point where pressure needs to be measured and other ends remains open to the atmosphere as shown in the figure. The tube generally contains mercury or any other liquid whose specific gravity is greater than the specific gravity of the liquid whose pressure is to be measured. Let us consider the following case where,

h_1 = height of the liquid above the datum line.

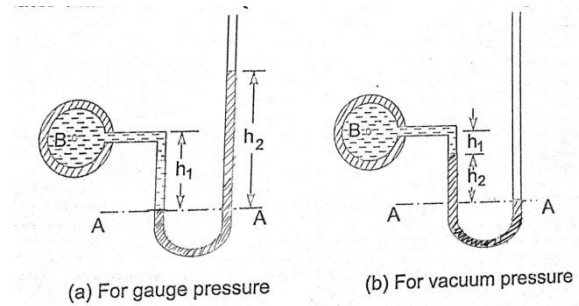
h_2 = height of the heavy liquid above the datum line

S_1 = Specific gravity of light liquid

S_2 = Specific gravity of heavy liquid

ρ_1 = density of light liquid = $1000 \times S_1$

ρ_2 = density of heavy liquid = $1000 \times S_2$



For gauge pressure let B be the point at which the pressure is to be measured. The value of pressure is P. The datum line is A-A.

Pressure above A-A in the left column = $P + \rho_1 \times g \times h_1$

Pressure above A-A in the right column = $\rho_2 \times g \times h_2$

Hence equating the two pressures,

$$P + \rho_1 \times g \times h_1 = \rho_2 \times g \times h_2$$

$$\therefore P = \rho_2 \times g \times h_2 - \rho_1 \times g \times h_1$$

For vacuum pressure level of heavy liquid in the manometer will be shown as figure -b

Pressure above A-A in the left column = $P + \rho_1 \times g \times h_1 + \rho_2 \times g \times h_2$

Pressure above A-A in the right column = 0

Hence equating the two pressures,

$$P + \rho_1 \times g \times h_1 + \rho_2 \times g \times h_2 = 0$$

$$\therefore P = - (\rho_2 \times g \times h_2 + \rho_1 \times g \times h_1)$$

SINGLE COLUMN MANOMETER:

Single column manometer is a modified form of U-tube manometer in which has a reservoir, having a large cross-sectional area about 100 times as compared to the area of the tube connected to one of the two limbs of the manometer. Due to large cross-sectional area of the reservoir, for any variation of pressure, the change in the liquid level in the reservoir will be very small which may be neglected. Hence the pressure is given by the height of the liquid in the other limb. The other limb may be vertical or inclined.

1. Vertical single column Manometer
2. Inclined single column Manometer

VERTICAL SINGLE COLUMN MANOMETER:

Figure shows a vertical single column manometer. Let X-X is the datum of the reservoir and right limb of manometer, when it is not connected to the pipe. When the manometer is

connected to the pip, due to high pressure at A, the heavy liquid in the reservoir will be pushed downward and will rise in the right limb.

h_1 = height of the center of pipe above X-X

h_2 = height of the heavy liquid above the datum line in right limb

S_1 = Specific gravity of light liquid

S_2 = Specific gravity of heavy liquid

ρ_1 = density of light liquid = $1000 \times S_1$

ρ_2 = density of heavy liquid = $1000 \times S_2$

Δh = fall of heavy liquid in the reservoir.

A = Cross-sectional area of the reservoir

a = Cross-sectional area of the right limb.

P is the pressure at point A to be measured.

Fall of heavy liquid in the reservoir will cause a rise in the heavy liquid level in the right limb.

So, $A \times \Delta h = a \times h_2$

$$\Delta h = \frac{a \times h_2}{A} \dots\dots\dots i$$

Now consider Y-Y as shown in the figure. Then the pressure in the right limb above Y-Y is

$$= \rho_2 \times g \times (\Delta h + h_2)$$

Pressure in the left limb above Y-Y is

$$= \rho_1 \times g \times (\Delta h + h_1) + P$$

Equating the pressures Pressure in the LL = Pressure in the RL

$$\rho_1 \times g \times (\Delta h + h_1) + P = \rho_2 \times g \times (\Delta h + h_2)$$

$$P = \rho_2 \times g \times (\Delta h + h_2) - \rho_1 \times g \times (\Delta h + h_1)$$

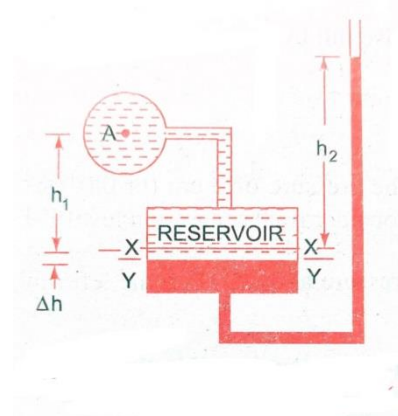
$$P = \Delta h(\rho_2 \times g - \rho_1 \times g) + \rho_2 \times g \times h_2 - \rho_1 \times g \times h_1$$

From equation (i) Putting the value of Δh

$$P = \frac{a \times h_2}{A}(\rho_2 \times g - \rho_1 \times g) + \rho_2 \times g \times h_2 - \rho_1 \times g \times h_1$$

When the area A is very large as compared to a, then $\frac{a}{A} \cong$ very small, so can be neglected.

$$\text{Then } P = \rho_2 \times g \times h_2 - \rho_1 \times g \times h_1$$



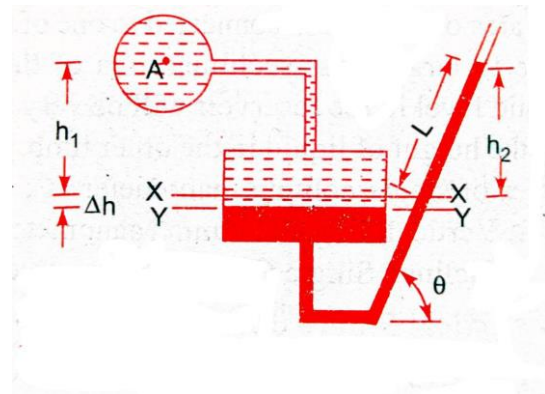
INCLINED SINGLE COLUMN MANOMETER:

This manometer is more sensitive; due to the inclination the distance moved by the heavy liquid in the right limb will be more.

L = Length of heavy liquid moved in right limb from X-X

θ = Inclination of right limb with horizontal

h_2 = Vertical rise of heavy liquid in the right limb from X-X = $L \times \sin \theta$



So, the pressure in the pipe becomes

$$P = \frac{a \times h_2}{A} (\rho_2 \times g - \rho_1 \times g) + \rho_2 \times g \times h_2 - \rho_1 \times g \times h_1$$

$$P = \frac{a \times L \times \sin \theta}{A} (\rho_2 \times g - \rho_1 \times g) + \rho_2 \times g \times L \times \sin \theta - \rho_1 \times g \times h_1$$

In case of very large A

$$P = \rho_2 \times g \times L \times \sin \theta - \rho_1 \times g \times h_1$$

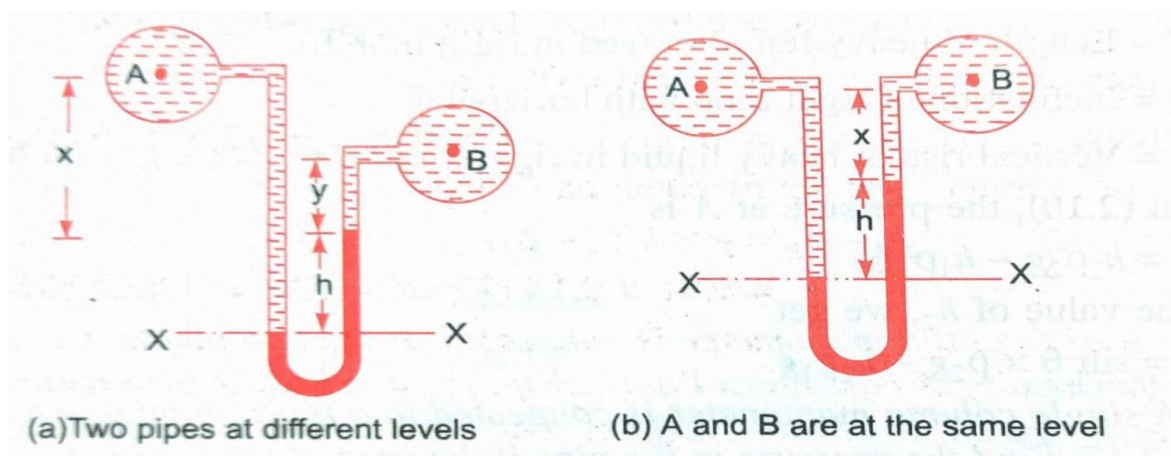
DIFFERENTIAL MANOMETER

Differential manometers are the devices used for measuring the differences of pressures between two points in a pipe or in two different pipes. A differential manometer consists of a U-tube, containing heavy liquid, whose two ends are connected to the point, whose pressure difference is to be measured. Differential manometers are:

1. U-tube differential manometer
2. Inverted U-tube differential manometer

U-TUBE DIFFERENTIAL MANOMETER

In the figure of differential manometer connected to the two pipes having different fluid of different specific gravity and at different levels A and B. Let the pressure in the pipes are P_A and P_B respectively.



Let, h = Differential of mercury level in the U-tube

x = Distance of center of A from the mercury level in the right limb

y = Distance of center of B from the mercury level in the right limb

S_1 = Specific gravity of liquid A

S_2 = Specific gravity of liquid B

ρ_1 = density of liquid A = $1000 \times S_1$

ρ_2 = density of liquid B = $1000 \times S_2$

ρ_g = density of heavy liquid

Taking the datum line at X-X

Pressure in the left limb above X-X = $P_A + \rho_1 \times g \times (h + x)$

Pressure in the right limb above X-X = $P_B + \rho_g \times g \times h + \rho_2 \times g \times y$

Equating pressure at two points

$$P_A + \rho_1 \times g \times (h + x) = P_B + \rho_g \times g \times h + \rho_2 \times g \times y$$

$$= P_A - P_B = \rho_g \times g \times h + \rho_2 \times g \times y - \rho_1 \times g \times (h + x)$$

$$= P_A - P_B = \rho_g \cdot g \cdot h + \rho_2 \cdot g \cdot y - \rho_1 \cdot g \cdot h - \rho_1 \cdot g \cdot x$$

$$= P_A - P_B = hg(\rho_g - \rho_1) + \rho_2 g y - \rho_1 g x$$

If the level A and B are at same level, then $x = y$ (consider this as x)

Pressure difference becomes

$$P_A - P_B = hg(\rho_g - \rho_1) + (\rho_2 - \rho_1)g x$$

If both the pipe contains liquid of same specific gravity $\rho_1 = \rho_2$

$$P_A - P_B = hg(\rho_g - \rho_1)$$

INVERTED U-TUBE DIFFERENTIAL MANOMETER

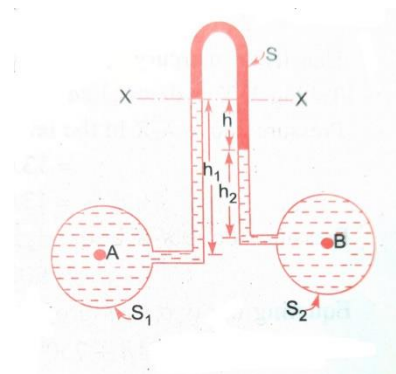
It consists of an inverted U-tube, containing a light liquid. The two ends of the tube are connected to the points whose difference of pressure is to be measured. It is used for measuring difference of low pressures.

Let, h = Difference of light liquid

h_1 = Height of liquid in left limb below the datum line X-X

h_2 = Height of liquid in right limb

S_1 = Specific gravity of liquid A



S_2 = Specific gravity of liquid B

ρ_1 = density of liquid A = $1000 \times S_1$

ρ_2 = density of liquid B = $1000 \times S_2$

ρ_s = density of light liquid

Let the pressure in the pipes are P_A and P_B respectively.

Taking the datum line at X-X

Pressure in the left limb below X-X = $P_A - \rho_1 \times g \times h_1$

Pressure in the right limb below X-X = $P_B - \rho_s \times g \times h - \rho_2 \times g \times h_2$

Equating the two pressure

$$P_A - \rho_1 \times g \times h_1 = P_B - \rho_s \times g \times h - \rho_2 \times g \times h_2$$

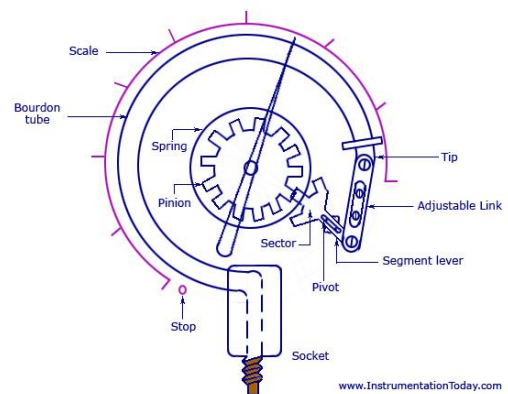
$$P_A - P_B = \rho_1 \times g \times h_1 - \rho_s \times g \times h - \rho_2 \times g \times h_2$$

MECHANICAL GAUGES

BOURDON TUBE PRESSURE GAUGE:

Bourdon Tubes are known for its very high range of differential pressure measurement in the range of almost 100,000 psi (700 MPa). It is an elastic type pressure transducer.

The device was invented by Eugene Bourdon in the year 1849. The basic idea behind the device is that, cross-sectional tubing when deformed in any way will tend to regain its circular form under the action of pressure. The bourdon pressure gauges used today have a slight elliptical cross-section and the tube is generally bent into a C-shape or arc length of about 27 degrees. The detailed diagram of the bourdon tube is shown in the figure.



Bourdon Tube Pressure Gauge

As seen in the figure, the pressure input is given to a socket which is soldered to the tube at the base. The other end or free end of the device is sealed by a tip. This tip is connected to a segmental lever through an adjustable length link. The lever length may also be adjustable. The segmental lever is suitably pivoted and the spindle holds the pointer as shown in the figure. A hair spring is sometimes used to fasten the spindle of the frame of the instrument to provide necessary tension for proper meshing of the gear teeth and thereby freeing the system from the backlash. Any error due to friction in the spindle bearings is known as lost motion. The mechanical construction has to be highly accurate in the case of a Bourdon Tube Gauge. If we consider a cross-section of the tube, its outer edge will have a larger

surface than the inner portion. The tube walls will have a thickness between 0.01 and 0.05 inches.

As the fluid pressure enters the bourdon tube, it tries to be reformed and because of a free tip available, this action causes the tip to travel in free space and the tube unwinds. The simultaneous actions of bending and tension due to the internal pressure make a non-linear movement of the free tip. This travel is suitable guided and amplified for the measurement of the internal pressure. But the main requirement of the device is that whenever the same pressure is applied, the movement of the tip should be the same and on withdrawal of the pressure the tip should return to the initial point.

A lot of compound stresses originate in the tube as soon as the pressure is applied. This makes the travel of the tip to be non-linear in nature. If the tip travel is considerably small, the stresses can be considered to produce a linear motion that is parallel to the axis of the link. The small linear tip movement is matched with a rotational pointer movement. This is known as multiplication, which can be adjusted by adjusting the length of the lever. For the same amount of tip travel, a shorter lever gives larger rotation. The approximately linear motion of the tip when converted to a circular motion with the link-lever and pinion attachment, a one-to-one correspondence between them may not occur and distortion results. This is known as angularity which can be minimized by adjusting the length of the link.

Other than C-type, bourdon gauges can also be constructed in the form of a helix or a spiral. The types are varied for specific uses and space accommodations, for better linearity and larger sensitivity. For thorough repeatability, the bourdon tubes materials must have good elastic or spring characteristics. The surrounding in which the process is carried out is also important as corrosive atmosphere or fluid would require a material which is corrosion proof. The commonly used materials are phosphor-bronze, silicon-bronze, beryllium-copper, inconel, and other C-Cr-Ni-Mo alloys, and so on.